AVENUE PRIMARY SCHOOL

PROGRESSION THROUGH CALCULATIONS
- A GUIDANCE DOCUMENT

SUMMER 2015

Progression through Calculation

Mission Statement

‘Expect the Best …To be the Best’

School Aims:

- To ensure that all pupils whatever their race, gender, age or ability feel valued and supported to achieve their best
- To promote positive learning attitudes and behaviour and create safe and effective learning environments where all children and staff show mutual respect for one another.
- To provide effective and strategic leadership at all levels.
- To deliver high standards of teaching that enable children to make progress and reach high standards of attainment.

Progression through Calculations – A Guidance Document

Aims

The overall aim is that by the end of year 6 children;

- have a secure knowledge of number facts and a good understanding of the four operations;
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;

This policy exemplifies a recommended progression through the four operations, beginning in Foundation Stage and carrying on to Year 6, and includes a selection of mental strategies. Teachers need to refer to the statutory requirements as set out in the National Curriculum Mathematics programmes of study: Key Stages 1 and 2 and the EYFS Framework.

The principal focus of mathematics teaching in Key Stage 1 is to ensure that pupils develop confidence and mental fluency with whole numbers. In Key Stage 2 children are encouraged to develop efficient written and mental methods.

When do children need to start recording?

Reception	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**	**Year 6**

Mental methods and jottings

Informal written methods

Developing formal written methods

It is important to encourage children to look first at the problem and then get them to decide which is the best method to choose – pictures, mental calculation with or without jottings or a structured recording.

Should children be taught one standard method for each operation?
Children should work through the school's agreed progression in order that they know and understand a compact standard method for each numerical operation by the end of Year 6.

What about children at different stages of attainment?
In many classes, children will be at different stages in their move towards efficiency. This process should not be rushed; children should be moved on when they are ready.

How can children's readiness for written calculations be judged?
Judgements will need to be made as to whether pupils possess sufficient skills to progress. Different prerequisite skills are needed for each operation. A short list of criteria for readiness for written methods of **addition and subtraction** would include:

- Do children know addition and subtraction facts to 20?
- Do they understand place value and can they partition numbers into hundreds, tens and units?
- Do they use and apply the commutative and associative laws of addition?
- Can they add at least three 1-digit numbers mentally?
- Can they add and subtract any pair of 2-digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?

Corresponding criteria to indicate readiness to learn written methods for **multiplication and division** are:

- Do the children know their times tables up to 12 x 12 and corresponding division facts?
- Do they know the result of multiplying by 0 or 1?
- Do they understand place value?
- Do they understand 0 as a place holder?
- Can they multiply 2 and 3 digits mentally by 10 and 100?
- Can they use their knowledge of all the multiplication tables to approximate?
- Can they find products using multiples of 10?
- Do they use the commutative and associative laws of multiplication?
- Can they halve and double 2-digit numbers mentally?
- Can they use multiplication facts to derive mentally, other multiplication facts they don't know?
- Can they explain their mental strategies orally and record them using informal jottings?

Children should be equipped to decide when it is best to use a mental or written method based on the knowledge that they are in control of this choice as they are able to carry out these methods with confidence.

PROGRESSION THROUGH CALCULATIONS FOR ADDITION

MENTAL CALCULATIONS- ONGOING

These are a **selection** of mental calculation strategies:

Mental recall of number bonds

6 + 4 = 10 ☐ + 3 = 10

25 + 75 = 100 19 + ☐ = 20

Use of near doubles

6 + 7 = double 6 + 1 = 13

Addition using partitioning and recombining

34 + 45 = (30 + 40) + (4 + 5) = 79

Counting on or back in repeated steps of 1, 10, 100, 1000

86 + 57 = 143 (by counting on in tens and then in ones)

460 - 300 = 160 (by counting back in hundreds)

Add the nearest multiple of 10, 100 and 1000 and adjust

24 + 19 = 24 + 20 – 1 = 43

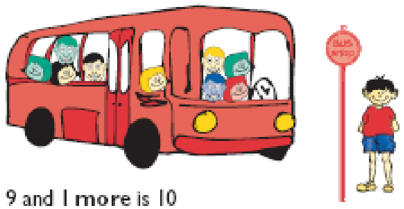
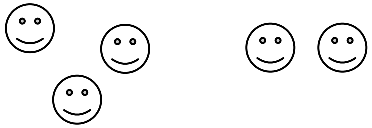
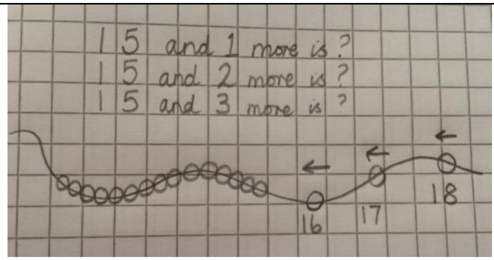
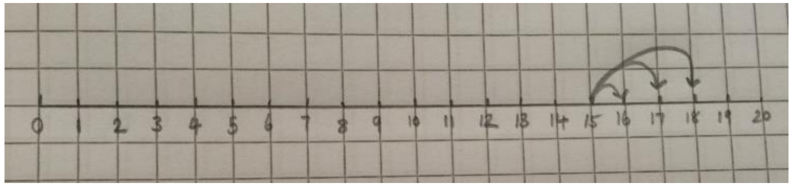
458 + 71 = 458 + 70 + 1 = 529

Use the relationship between addition and subtraction

36 + 19 = 55 19 + 36 = 55

55 – 19 = 36 55 – 36 = 19

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

<u>Foundation Stage</u> Children begin to add/count on mentally using rhymes and begin to record in the context of play or practical activities e.g. Recording with marks, stamps or objects Children use the language of 1 more by adding one to a group e.g. tower of cubes Adding stories and role play, encourages the use of language for addition. Children use large number tiles to identify one more. Children combine 2 groups of objects through cutting and sticking and picture representation of an addition sentence. They use practical resources to support calculation.	 9 and 1 more is 10 9 add 1 equals 10 9 + 1 = 10   3 and 2 makes 5
<u>Year 1</u> Children add by counting on. They first find 1 more, then count in steps of 1 within 20. The teacher should model drawing jumps on the numbered number line to support understanding of the mental method. Children can count on from the first number using fingers, objects, themselves etc.	 

Children learn that addition can be done in any order and are taught that it is more efficient to put the larger number first.

Children need to understand the concept of equality before using the = sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as the 'answer'.
E.g. 2 =1+1   and 2+3 = 4+1
Children begin to record addition number sentences using + and =.

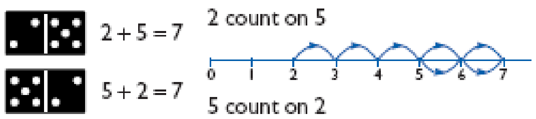
Missing numbers need to be placed in all possible places within the number sentence.
4+☐ = 7
☐ + 2= 8
Teachers should cover up operations as well as numbers.

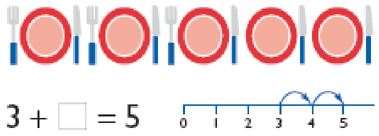
Use addition in terms of 'how many more' to calculate the difference.

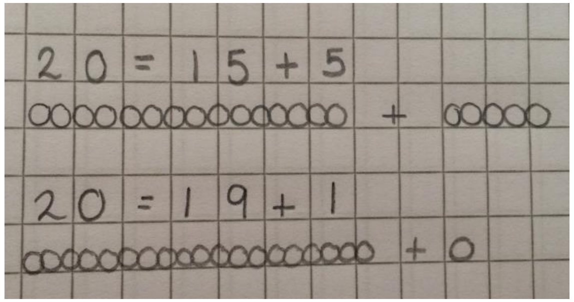
Children learn number bonds to 20.

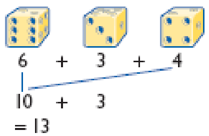
Children begin to add 3 single digit numbers, by looking for pairs of numbers or doubles to aid mental calculation. They begin to bridge through ten where necessary.

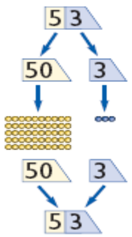
Children begin to learn place value of 2 digit numbers, to add in tens and ones.











Year 2

Children add 3 single digit numbers. They recall addition facts to 20 fluently and use related facts up to 100.

Children learn to count on in tens and ones on the number line to add a two-digit number and ones, a two-digit number and tens and two two-digit numbers.

They draw blank number lines and draw how many they are counting on.
Children can become more efficient by adding the units in one jump using known facts.

This can then be followed by adding the tens in one jump and the units in one jump.

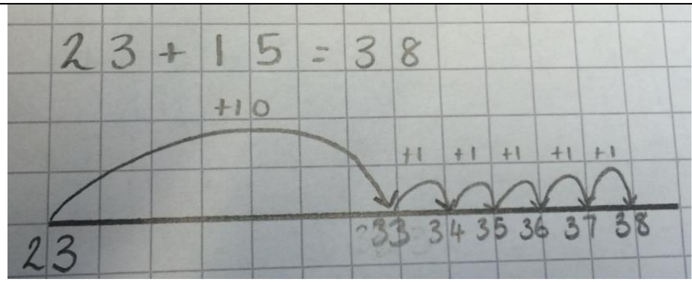
Children add 9 and 11 by adding 10 and adjusting by 1.

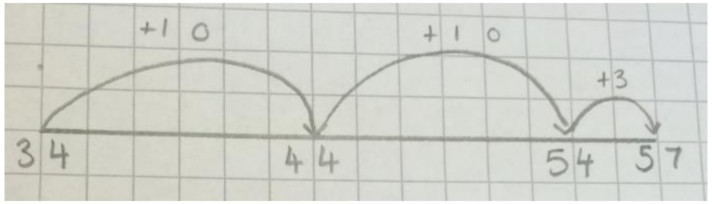
Continue with using a range of equations as in year 1, but with larger numbers such as multiples of 10.

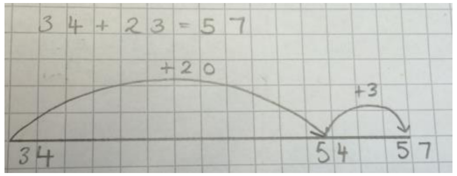
70 + $\square$ = 20 + $\square$

Children learn that subtraction is the inverse of addition and use known number facts to calculate mentally.

Children continue to add by bridging through 10 where necessary.
They show that addition of 2 numbers can be done in any order (commutative).



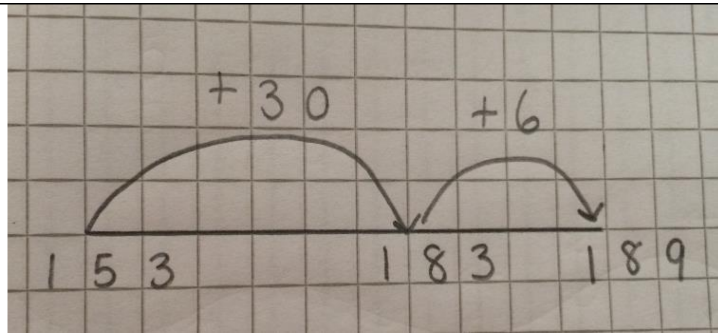




Year 3

Children count on from the largest number irrespective of the order of the calculation.

+ = signs and missing numbers
Continue using a range of equations as in Year 1 and 2, but with appropriate, larger numbers.



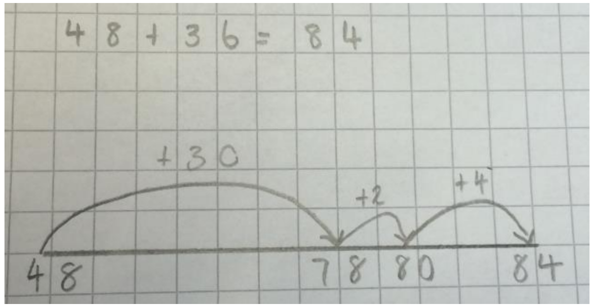
Partition into hundreds, tens and ones

- Partition numbers and recombine.
- Count on by partitioning the second number only e.g.

36 + 153 = 153 + 30 + 6
= 183 + 6
= 189

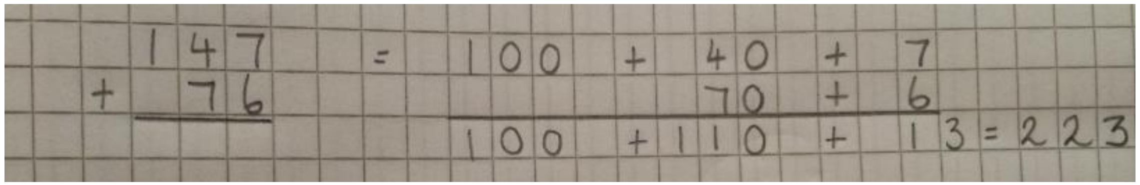
Add a near multiple of 10 to a two-digit number
Secure mental methods by using a number line to model the method. Continue as in Year 2, but with appropriate numbers e.g. 35 + 19 is the same as 35 + 20 – 1.

Children need to be secure adding multiples of 10 to any two-digit number including those that are not multiples of 10.

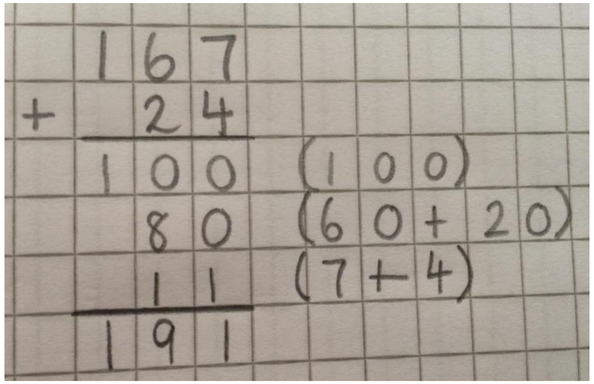


Children add ones, tens and hundreds to three-digit numbers using pencil and paper procedures leading to column addition.

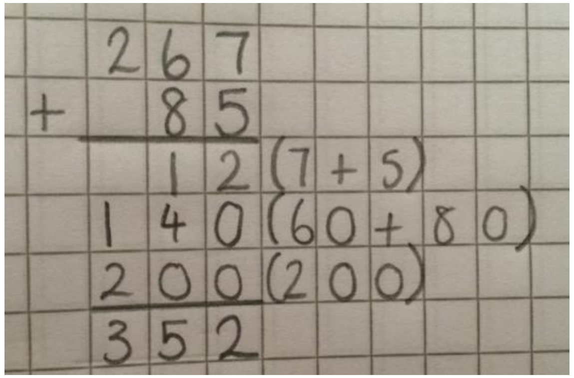
1. Horizontal expansion using partitioning in columns



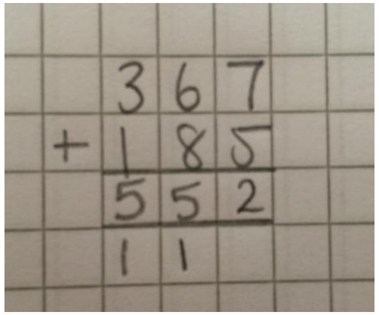
2. Vertical expansion in columns – adding most significant digits first



3. Vertical expansion in columns – adding least significant digits first in preparation for carrying



Column addition (including carrying) can be taught when children are ready to move on.
Children carry below the line.



<table>
<tr>
<td>
<u>Year 4</u>

+ = signs and missing numbers
Continue using a range of equations as in Year 1 and 2, but with appropriate numbers.

Partition into hundreds, tens and ones
Partition the second number only (as in Year 3 with appropriate numbers)

Add the nearest multiple of 10 or 100, then adjust
Continue as in Year 3, but with appropriate numbers e.g. 163 + 29 is the same as 163 + 30 – 1

<u>Pencil and paper procedures – column addition of numbers with up to 4 digits</u>

2245 + 186

Children should carry below the line.

Extend to decimals in the context of money ensuring children know that the decimal points should line up under each other, particularly when adding mixed amounts.
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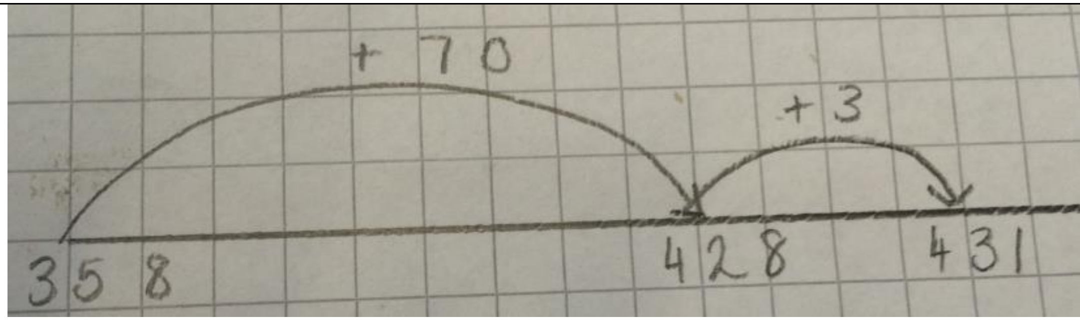

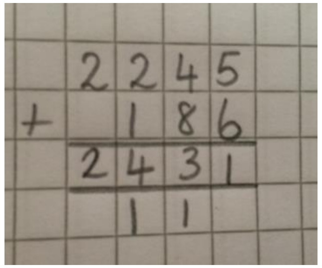

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<tr>
<td>
<u>Year 5</u>

+ = signs and missing numbers
Continue using a range of equations as in Year 1 and 2, but with appropriate numbers.

Add or subtract the nearest multiple of 10 or 100, then adjust
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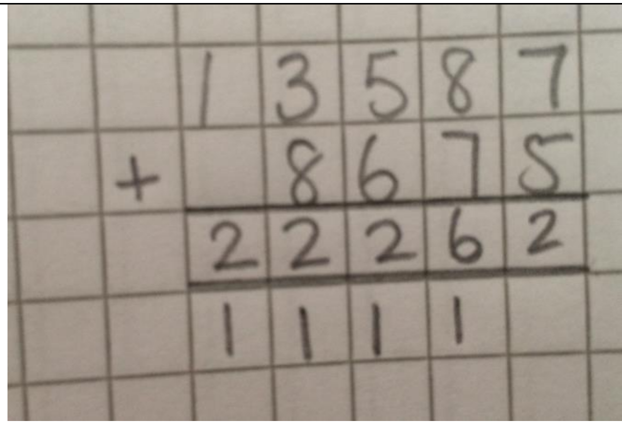
Continue as in Year 2, 3 and 4 but with appropriate numbers
e.g. 458 + 79 = is the same as 458 + 80 - 1

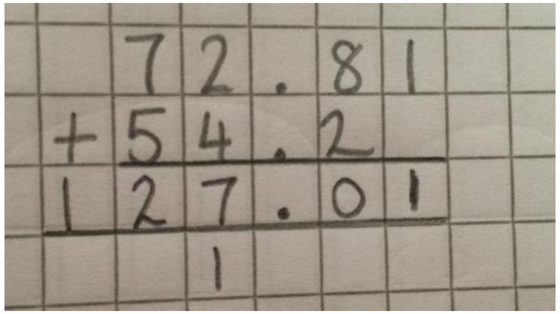
<u>Pencil and paper procedures – column addition with numbers with more than 4 digits</u>

13587 + 8675



Revert to expanded methods if the children experience any difficulty.
Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits).

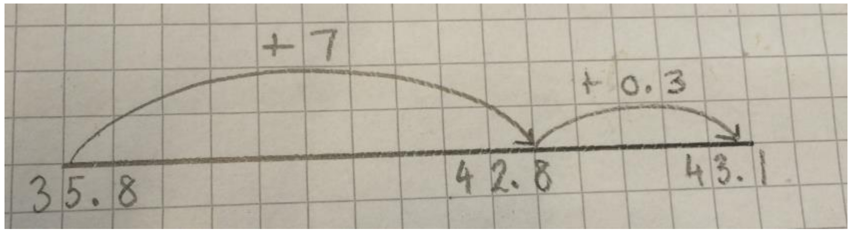
72.81 + 54.2



<u>Year 6</u>

+ = signs and missing numbers
Continue using a range of equations as in Year 1 and 2, but with appropriate numbers.

Partition into hundreds, tens, ones and decimal fractions
Either partition both numbers and recombine or partition the second number only e.g.
35.8 + 7.3 = 43.1



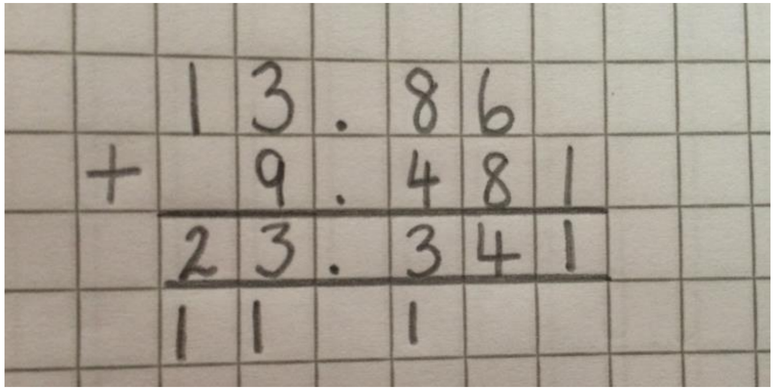
Add the nearest multiple of 10, 100 or 1000, then adjust
Continue as in Year 2, 3, 4 and 5, but with appropriate numbers including extending to adding 0.9, 1.9, 2.9 etc.

<u>Pencil and paper procedures with any number of digits and decimals with 1, 2 and/or 3 decimal places</u>

Children should also continue to practise addition using the formal written method of column addition with larger numbers.

13.86 + 9.481

Revert to expanded methods if the children experience any difficulty.



By the end of year 6, children will have a range of calculation methods, mental and written. This will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

1) **they are not ready.**

2) **they are not confident.**

Children should be encouraged to approximate their answers before calculating.
Children should be encouraged to check their answers after calculation using an appropriate strategy.
Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

PROGRESSION THROUGH CALCULATIONS FOR SUBTRACTION

MENTAL CALCULATIONS ONGOING

These are a **selection** of mental calculation strategies
(Please refer to NNS mental calculation strategy guidance and for Foundation stage; Numbers and patterns: laying foundations in mathematics, both on the staff drive)

Mental recall of addition and subtraction facts

10 – 6 = 4	17 - ☐ = 11
20 - 17 = 3	10 - ☐ = 2

Find a small difference by counting up
82 – 79 = 3

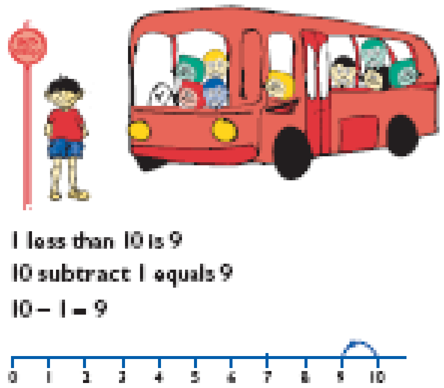
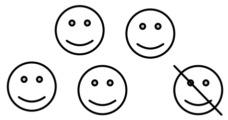
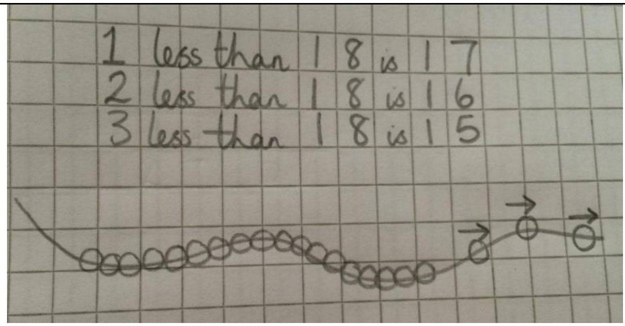
Counting on or back in repeated steps of 1, 10, 100, 1000
86 - 52 = 34 (by counting back in tens and then in ones)
460 - 300 = 160 (by counting back in hundreds)

Subtract the nearest multiple of 10, 100 and 1000 and adjust
24 - 19 = 24 - 20 + 1 = 5
458 - 71 = 458 - 70 - 1 = 387

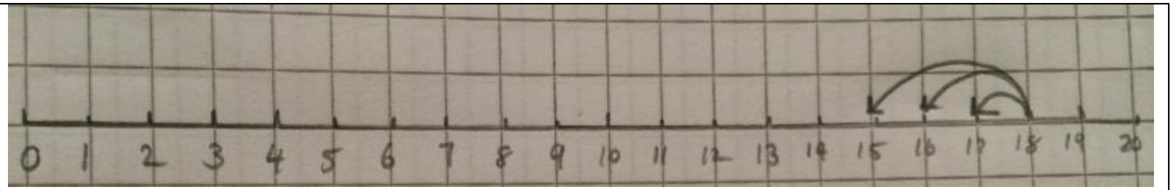
Use the relationship between addition and subtraction

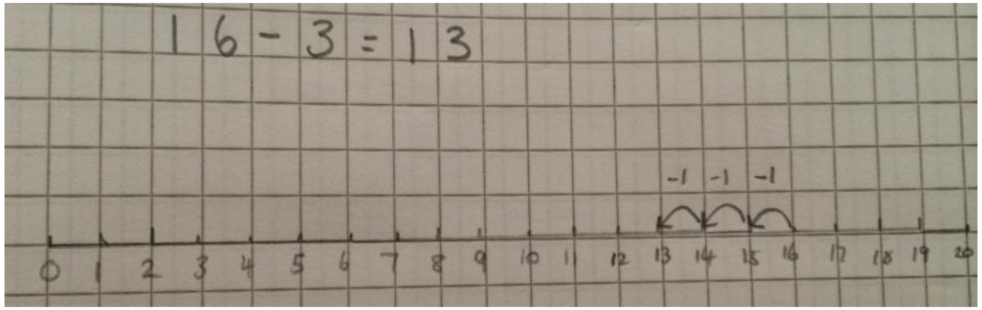
36 + 19 = 55	19 + 36 = 55
55 – 19 = 36	55 – 36 = 19

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

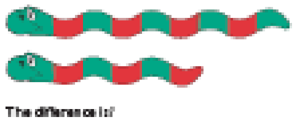
<u>Foundation stage</u> Children begin to record in the context of play or practical activities e.g. counting rhymes that count back. Children remove objects from a group ‘I have 5 apples and a take one away how many are left?’ Use the language of 1 less by taking 1 from a group e.g. tower of cubes In take away stories such as role play the use of language of subtraction is encouraged. Use large numbered floor tiles to identify one less. Children use models, images and picture representation of a subtraction sentence.	  5 take away 1 leaves 4
<u>Year 1</u> Children count back in steps of 1 then 10. They also identify missing numbers in a number line. They subtract one-digit and two-digit numbers to 20 including 0. Children first find 1 less then count back in steps of 1. Children can count back 1 from the first number using fingers, objects, themselves etc.	

Teacher should model drawing jumps on the numbered number line to support understanding of the mental method.





The number line should also be used to show that 16 - 3 means the ‘difference between
16 and 3’ or ‘the difference between 3 and 16’ and how many jumps they are apart.



Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

Children learn that subtraction must start with the larger number and count back the smaller number.

Children begin to record subtraction number sentences using - and =.

Missing numbers need to be placed in all possible places within the number sentence.

Children begin to subtract to solve simple word problems.

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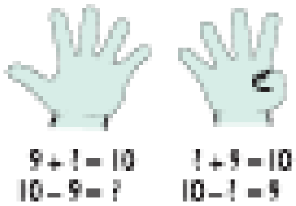
<td>
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<td><u>Year 2</u>

Counting back:

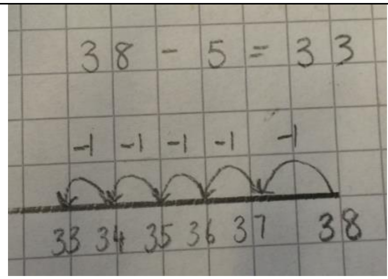
Children subtract ones from two-digit numbers, tens from two-digit numbers and a two-digit number from a two-digit number.

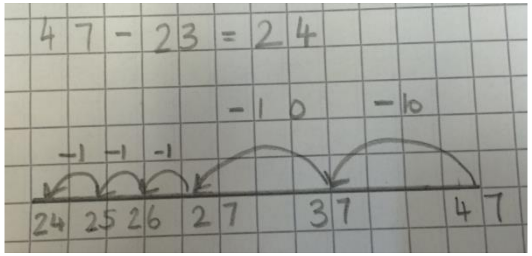
They draw blank number lines and draw how many they are counting back.

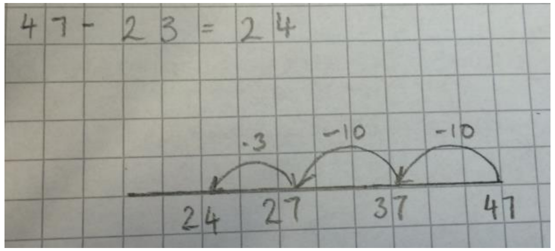
Children learn to count back in tens and ones on the number line.

Children become more efficient by subtracting the units in one jump (by using the known fact 7 – 3 = 4).

Then they subtract the tens in one jump and the units in one jump.

Bridging through ten can help children become more efficient.</td>
<td>

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42 – 25 = 17

Children subtract 9 and 11 by subtracting 10 and adjusting by 1 using the hundred square.
They subtract by bridging through 10 where necessary.

Continue with using a range of equations as in year 1, but with larger numbers such as multiples of 10.

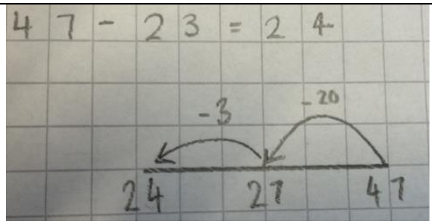
100 - $\square$ = 40

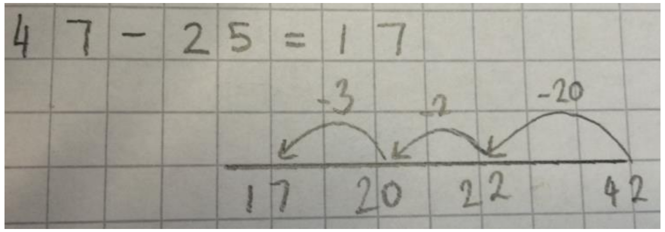
Children find the difference by counting on with larger numbers on the number line.

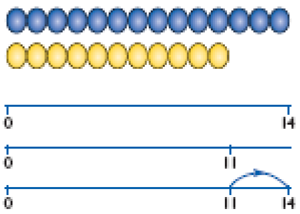
Children know that subtraction is the inverse of addition and use known number facts to calculate mentally.

Children begin to subtract larger 2 digit numbers by partitioning the second number only.

37 – 12 = 37 – 10 = 27
= 27 – 2
= 25







The difference between 11 and 14 is 3.
14 – 11 = 3
11 + $\square$ = 14

20 = 12 + 8 8 + 12 = 20
20 – 8 = 12 20 – 12 = 8
(10 + 2 + 8 = 10 + 10 = 20)

Year 3

Counting up:

- = signs and missing numbers
Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

Find a small difference by counting on
Continue as in Year 2, but with appropriate numbers e.g. 102 – 97 = 5

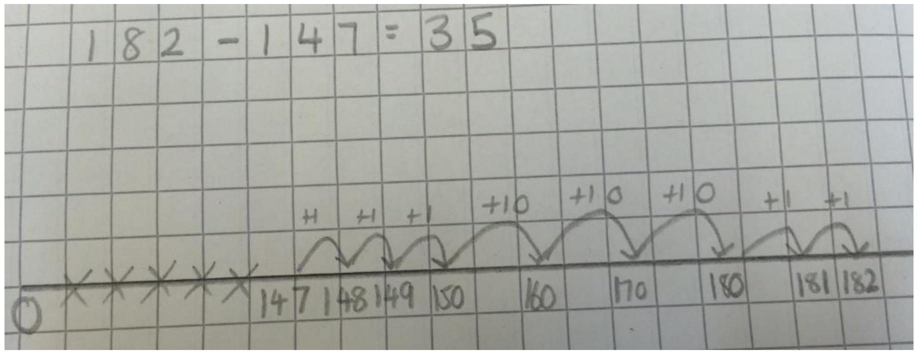
Subtract mentally a ‘near multiple of 10’ to or from a two-digit number.
Continue as in Year 2, but with appropriate numbers e.g. 78 – 49 is the same as 78 – 50 + 1

Use known number facts and place value to subtract
Continue as in Year 2, but with numbers with up to 3 digits e.g.197 – 15 = 182

Pencil and paper procedure using numbers with up to 3 digits:
If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on.

Count up from 147 to 182 in jumps of 10 and jumps of 1.

The number line should still show 0 so children can cross out the section from 0 to the smallest number. They then associate this method with ‘taking away’.



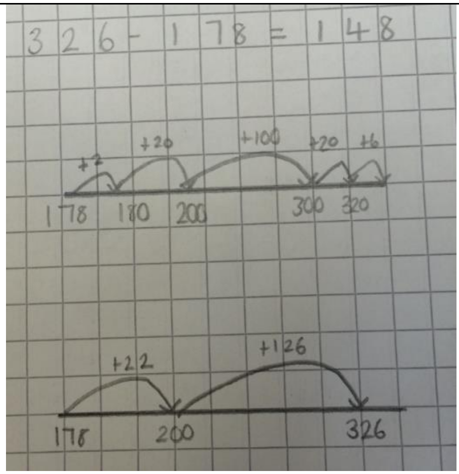
With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 – 12, 86 – 77 or 43 – 28.

Children will continue to use empty number lines, with increasingly large numbers.

With three-digit numbers the number of steps can again be reduced, provided that

children are able to work out answers to calculations such as 178 + ☐ = 200 and

200 + ☐ = 326 mentally.



Children subtract ones, tens and hundreds from three-digit numbers using pencil and paper procedures leading to column subtraction.

Partitioning**:**

This process should be demonstrated using arrow cards to show the partitioning and base 10 materials to show the partitioning of the number.

When solving the calculation 89 – 57, children should know that 57 **does NOT EXIST AS AN AMOUNT** it is what you are subtracting from the other number. Therefore, when using base 10 materials, children would need to count out only the 89.

189	=	100 +	80	+	9		
- 57			50	+	7		
		100 +	30	+	2	=	132

This would be recorded by the children as

Initially, the children will be taught using examples that do not need the children to exchange.

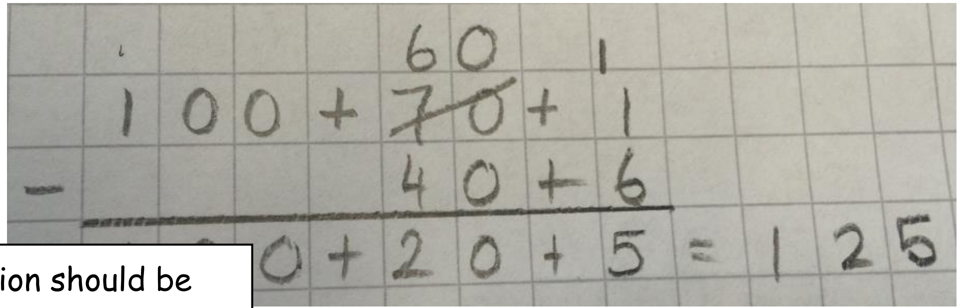
<u>From this the children will begin to exchange.</u>

```
  171
-  46
```

```
Step 1          100 + 70  +  1
              -       40  +  6

Step 2       100 + 60  +  11
           -       40  +   6
             100 + 20  +   5   =   125
```

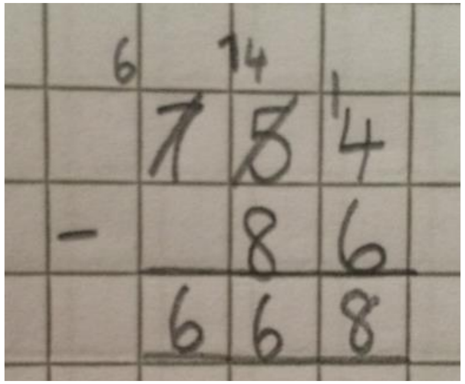
The calculation should be read as e.g. take 6 from 1.

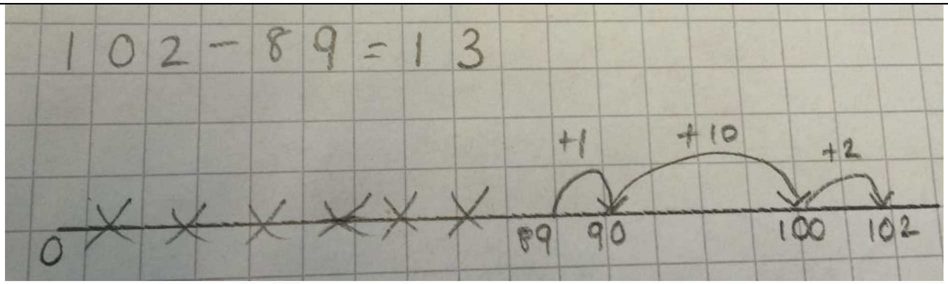
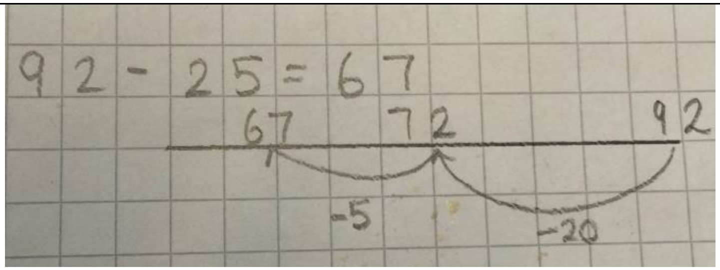


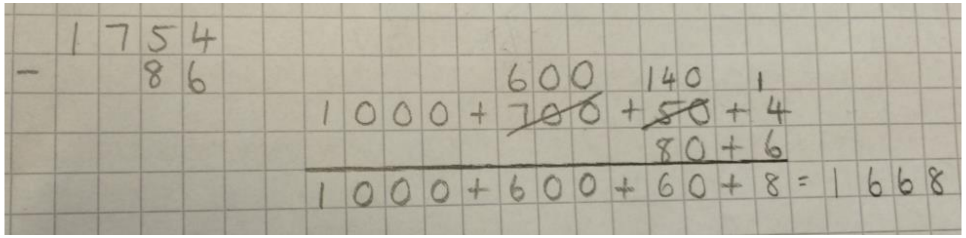
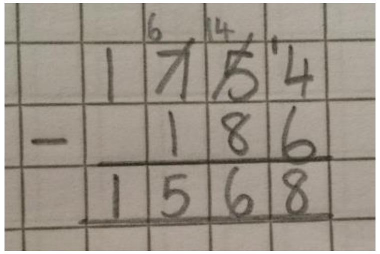
Children should know that units line up under units, tens under tens, and so on.

Column subtraction can be taught when children are ready.

754 - 86



Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.	
<u>Year 4</u> Find a small difference by counting up e.g. 5003 – 4996 = 7 This can be modelled on an empty number line (see below). Children should be encouraged to use known number facts to reduce the number of steps. Subtract the nearest multiple of 10, then adjust. Continue as in Year 2 and 3, but with appropriate numbers. Use known number facts and place value to subtract	

<u>Pencil and paper procedures – column subtraction using numbers with up to 4 digits.</u> Consider if children are secure in column subtraction from Year 3 and can apply this to 4-digit numbers. If not, partitioning should be continued: $\begin{array}{r} 1754 \\ -\quad 86 \\ \hline \end{array}$ Step 1 $\begin{array}{r} 1000 + 700 + 50 + 4 \\ - \qquad\qquad 80 + 6 \\ \hline \end{array}$ Step 2 1000 + 700 + 40 + 14 *(adjust from T to U)* $\begin{array}{r} - \qquad\qquad 80 + 6 \\ \hline \end{array}$ Step 3 1000 + 600 + 140 + 1 *(adjust from H to T)* $\begin{array}{r} - \qquad\qquad 80 + 6 \\ \hline \end{array}$ 1000 + 600 + 60 + 8 = 1668 Column subtraction: 1754 – 186	This would be recorded by the children as  

Year 5

Find the difference by counting up:

e.g. 8006 – 2993 = 5013
This can be modelled on an empty number line.

Subtract the nearest multiple of 10 or 100, then adjust.
Continue as in Year 2, 3 and 4, but with appropriate numbers.

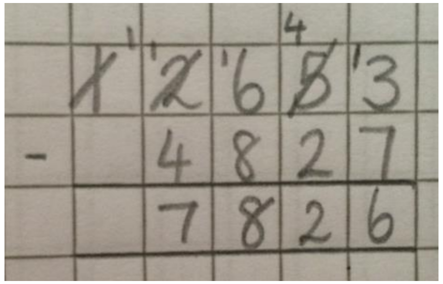
Use known number facts and place value to subtract decimal numbers with 1dp and compliments of 1.

6.1 – 2.4 = 3.7

Pencil and paper procedures – column subtraction using numbers with more than 4 digits:

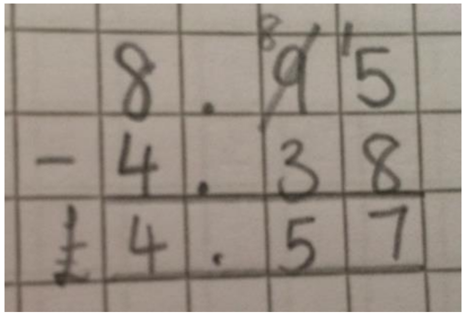
If your children have reached the concise stage of decomposition then they will continue this method through into years 5 and 6. They will not go back to using the expanded methods.

Column subtraction:
12653 - 4827



Children should:

- ✓ *be able to subtract numbers with different numbers of digits;*
- ✓ *using this method, children should also begin to find the difference between two three-digit sums of money, with or without 'adjustment' from the pence to the pounds;*
- ✓ *know that decimal points should line up under each other.*



Year 6 Find the difference by counting up e.g. 8000 – 2785 = 5215 To make this method more efficient, the number of steps should be reduced to a minimum through children knowing: ▪ Complements to 1, involving decimals to two decimal places (0.16 + 0.84) ▪ Complements to 10, 100 and 100 Subtract the nearest multiple of 10, 100 or 1000, then adjust Continue as in Year 2, 3, 4 and 5 but with appropriate numbers. Use known number facts and place value to subtract decimals with different numbers of decimal places 0.5 – 0.31 = 0.19 Pencil and paper procedures: Children should practise subtraction using the formal written method of column subtraction with larger numbers (see Year 5).	

By the end of year 6, children will have a range of calculation methods, mental and written. This will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

1) **they are not ready.**
2) **they are not confident.**

Children should be encouraged to approximate their answers before calculating.

Children should be encouraged to check their answers after calculation using an appropriate strategy.
Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

PROGRESSION THROUGH CALCULATIONS FOR MULTIPLICATION

MENTAL CALCULATIONS (ONGOING)

These are a **selection** of mental calculation strategies:
Doubling and halving
Applying the knowledge of doubles and halves to known facts.
e.g. 8 x 4 is double 4 x 4

Using multiplication facts
Tables should be taught every day from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the day.

Year 2	2, 5 and 10 times table
Year 3	2, 3, 4, 5, 8 and 10 times table
Year 4	Derive and recall all multiplication facts up to 12 x 12
Years 5 & 6	Derive and recall quickly all multiplication facts up to 12 x 12.

Using and applying division facts
Children should be able to utilise their tables knowledge to derive other facts.
e.g. If I know 3 x 7 = 21, what else do I know?
30 x 7 = 210, 300 x 7 = 2100, 3000 x 7 = 21 000, 0.3 x 7 = 2.1 etc
Use closely related facts already known
13 x 11 = (13 x 10) + (13 x 1)
= 130 + 13
= 143
Multiplying by 10 or 100
Knowing that the effect of multiplying by 10 is a shift in the digits one place to the left.

Knowing that the effect of multiplying by 100 is a shift in the digits two places to the left.

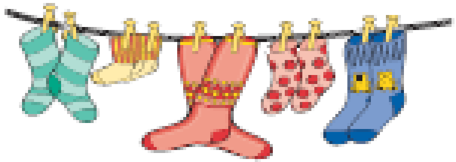
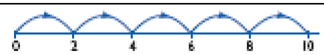
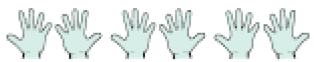
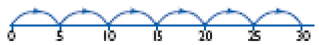
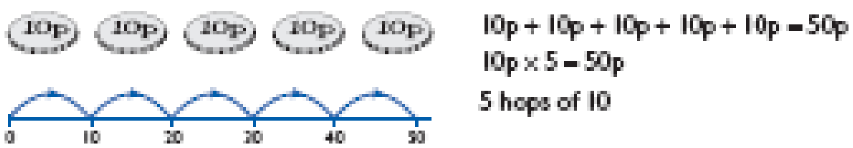
Partitioning

23 x 4 = (20 x 4) + (3 x 4)
= 80 + 12
= 102

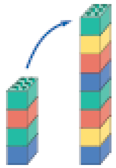
Use of factors

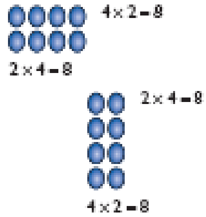
8 x 12 = 8 x 4 x 3

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

<u>Foundation stage</u> Children begin to count in groups of 2, 5 and 10 using objects, recite counting, songs and rhymes. They count related groups of the same size in games and practical activities. Links are also made to problem solving activities.	
<u>Year 1</u> Children group objects in 2, 5 and 10. Children start to use visual images as repeated addition. 2 + 2 + 2 + 2 + 2 = 10 Model this as jumps on a number line. Practically double numbers to 10 and link this with multiplying by 2.	   

<table>
<tr>
<td>Begin to show visual representation of this using an array.

Children solve practical problems involving multiplication such as;
There are 4 bikes. Each bike has 2 wheels, how many wheels is that?</td>
<td>
double 4 is 8
4 × 2 = 8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<tr>
<td><u>Year 2</u>

Children use repeated addition number sentences to calculate multiplication;

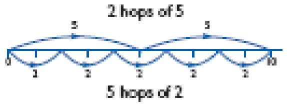
4x3 = 3+3+3+3

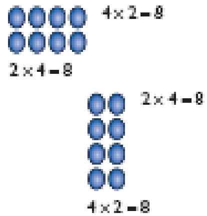
Children are taught to calculate multiplication questions by jumping in groups on a number line.
Children explore the fact that multiplication, like addition, can be done in any order.

Children continue to show visual representation of this using an array.

Children begin to record multiplication number sentences using x and =.

They are then taught to develop an understanding of the families of numbers to work out the missing numbers e.g.

7 x 2 = □   □ = 2 x 7</td>
<td>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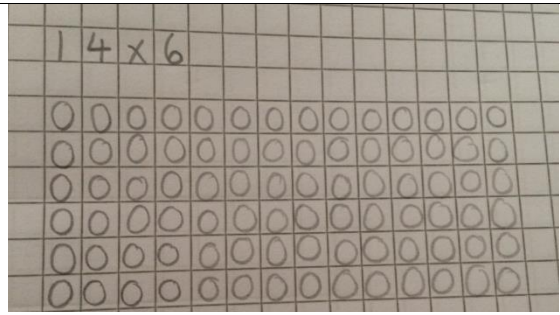
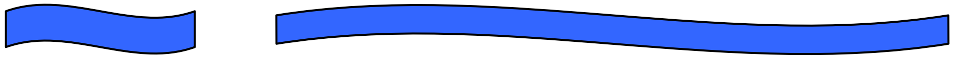
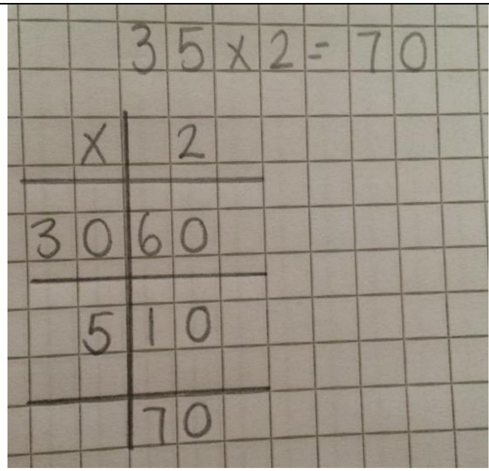
7 x □ = 14 14 = □ x 7 □ x 2 = 14 14 = 2 x □ □ x ∇ = 14 14 = □ x ∇ Children use multiplication to solve more complex word problems.	
<u>Year 3</u> x = signs and missing numbers Continue using a range of equations as in Year 2, but with appropriate numbers. <u>Arrays and repeated addition</u> Continue to understand multiplication as repeated addition (using the number line and bead bars) and continue to use arrays (as in Year 2). 14 x 6 <u>Doubling multiples of 5 up to 50</u> 35 x 2 = 70 Children will also develop an understanding of <u>Scaling</u> e.g. Find a ribbon that is 4 times as long as the blue ribbon Children use symbols to stand for unknown numbers to complete equations using inverse operations □ x 5 = 20 3 x △ = 18 □ x O = 32 <u>Pencil and paper procedures – partitioning to multiply a two-digit number by ones.</u> 38 x 5 = (30 x 5) + (8 x 5)	  5cm 20cm

= 150 + 40
= 190

Leading to partitioning using the grid method:

34 x 2

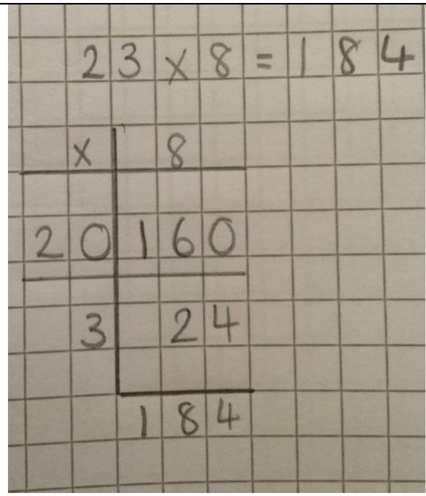
Children use known facts and place value to carry out simple multiplications.



Year 4

Children continue to use arrays if not ready for the grid method of multiplication.

Pencil and paper procedures - short multiplication, multiplication of a two and three- digit numbers by a single digit number



23 x 8

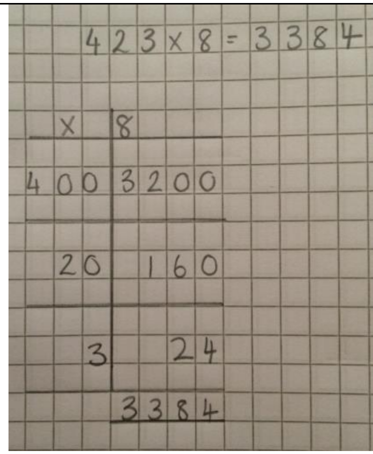
Leading to:

Leading to: expanded short multiplication:

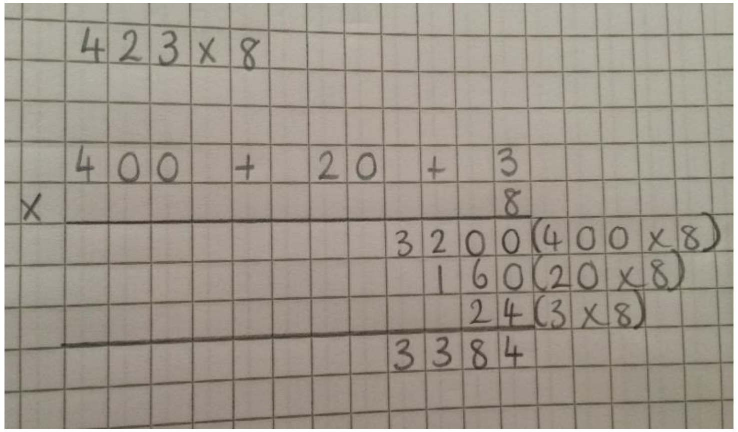
The next step is to move the number being multiplied to an extra row at the top. Presenting the grid this way helps children to set out the addition of the partial products.

The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.

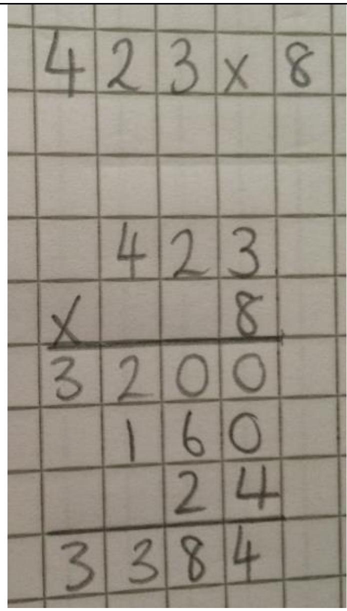
Children should describe what they do by referring to the actual values of the digits in the columns. Most children should be able to use this expanded method for HTU × U by the end of Year 4.



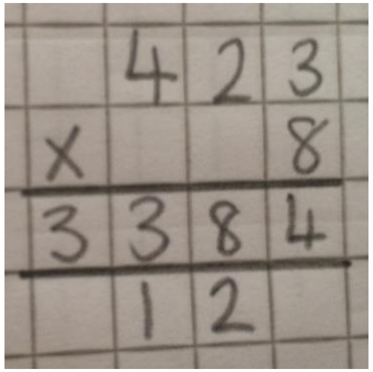
423 x 8



Leading to:



Leading to:

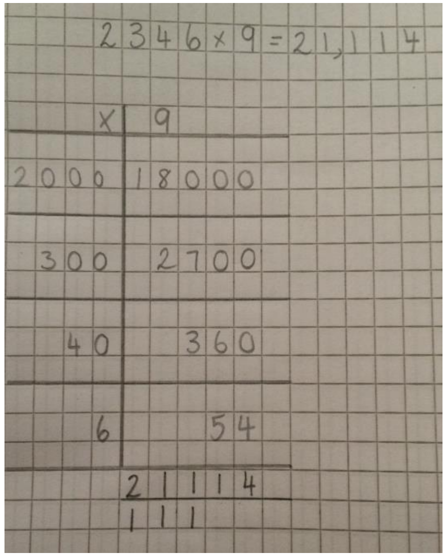


<u>Year 5</u>

<u>Pencil and paper procedures:</u>

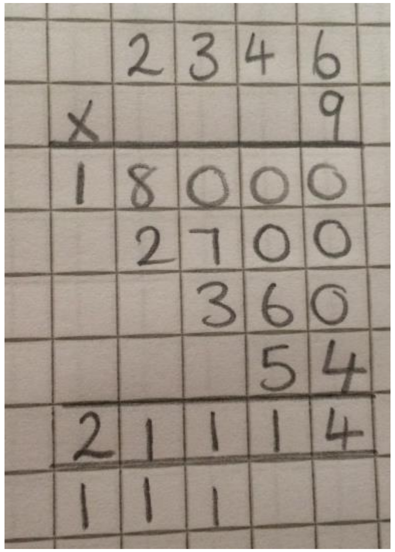
<u>Short multiplication – a four-digit number multiplied by a single digit</u>

Grid method:

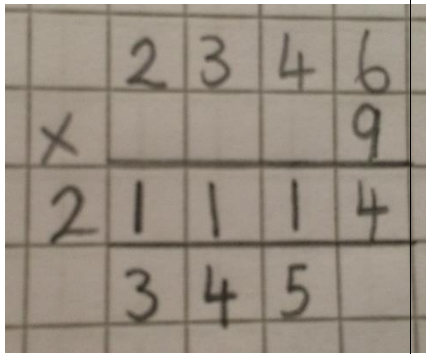


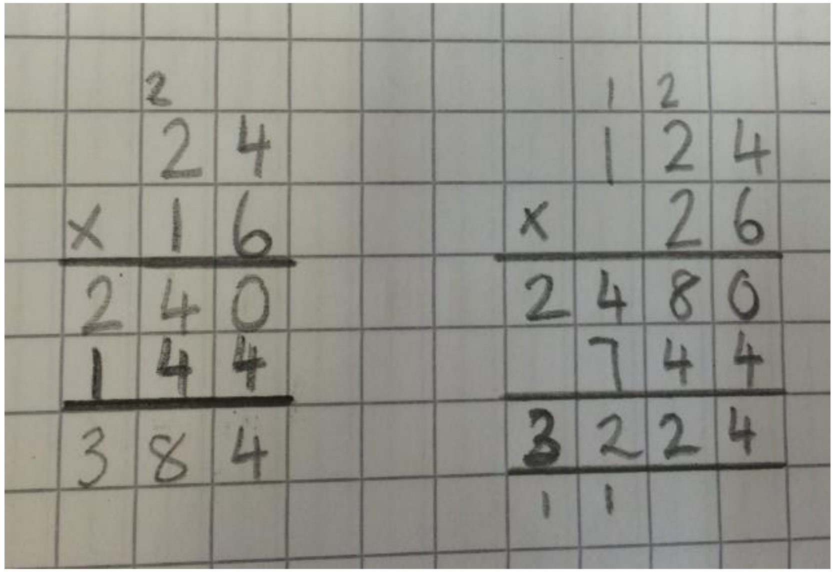
Long multiplication – multiplication of a two-digit or a three-digit number by a two-digit number:

Leading to:



Leading to:



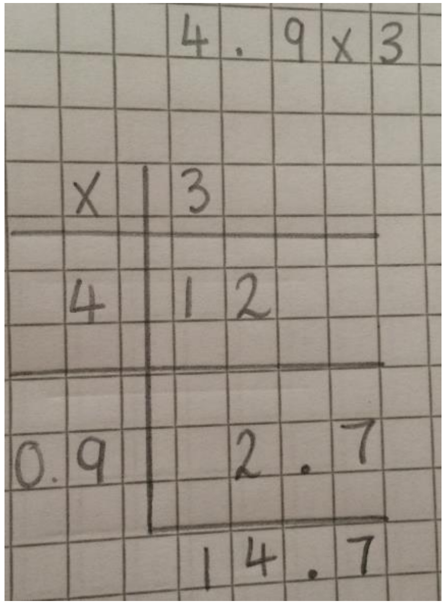


The aim is for most children to use this long multiplication method for HTUxTU by the end of year 5.

Using similar methods, they will be able to multiply decimals with one decimal place by a single digit number, approximating first. They should know that the decimal points line up under each other.

e.g. 4.9 x 3

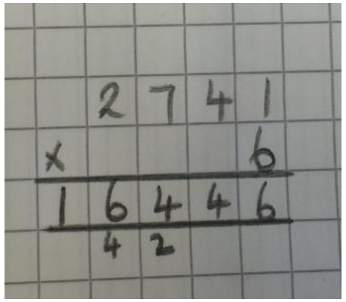
Children will approximate first
4.9 x 3 is approximately 5 x 3 = 15



YEAR 6

ThHTU x TU
Long multiplication – multiplication of a four-digit number by a two-digit number

ThHTU x U
Short multiplication – multiplication of a four-digit number by a single digit



Extend to decimals with up to two decimal places.

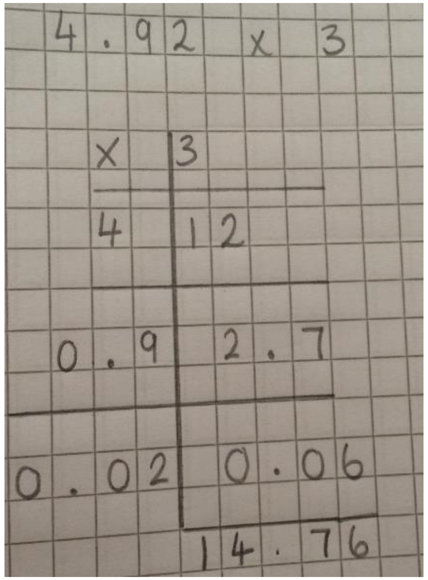
Children who are already secure with multiplication for HTU × U and HTU × TU should have little difficulty in using the same method for applying decimals.

Using similar methods, they will be able to multiply decimals with up to two decimal places by a single digit number and then two digit numbers, approximating first. They should know that the decimal points line up under each other.

For example:

4.92 x 3

Children will approximate first
4.92 x 3 is approximately 5 x 3 = 15



By the end of year 6, children will have a range of calculation methods, mental and written. This will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

1) **they are not ready.**
2) **they are not confident.**

Children should be encouraged to approximate their answers before calculating.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

PROGRESSION THROUGH CALCULATIONS FOR DIVISION

MENTAL CALCULATIONS

These are a **selection** of mental calculation strategies:

Doubling and halving
Knowing that halving is dividing by 2

Deriving and recalling division facts
Tables should be taught every day from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the day.

Year 2	2, 5 and 10 times table
Year 3	2, 3, 4, 5, 8 and 10 times table
Year 4	Derive and recall division facts for all tables up to 12 x 12
Year 5 & 6	Derive and recall quickly division facts for all tables up to 12 x 12

Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts.
e.g. If I know 3 x 7 = 21, what else do I know?
30 x 7 = 210, 300 x 7 = 2100, 3000 x 7 = 21 000, 0.3 x 7 = 2.1 etc

Dividing by 10 or 100
Knowing that the effect of dividing by 10 is a shift in the digits one place to the right.
Knowing that the effect of dividing by 100 is a shift in the digits two places to the right.

Use of factors

378 ÷ 21	378 ÷ 3 = 126 126 ÷ 7 = 18	378 ÷ 21 = 18

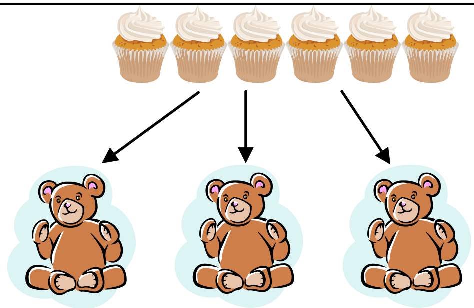
Use related facts
Given that 1.4 x 1.1 = 1.54
What is 1.54 ÷ 1.4, or 1.54 ÷ 1.1?

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHOD

<table>
<tr>
<td><u>Foundation stage</u>

Practical division as grouping e.g. buttons, beads etc

Children share objects practically into equal groups e.g;
“Share the cakes between the three bears. How many cakes will they each have?”

Links are made to problem solving activities.</td>
<td></td>
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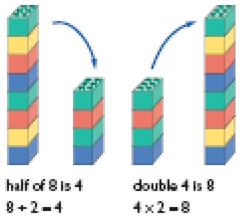
<tr>
<td><u>Year 1</u>

Halving to match doubling and understand it is the opposite.

Sort a set of objects by grouping equally into 2’s, 3’s, 4’s etc.

Children begin to use practical grouping to solve word problems.</td>
<td>
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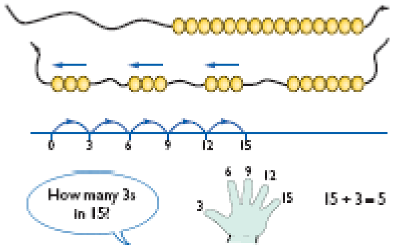
e.g. "There are 12 daffodil bulbs. Plant 3 in each pot. How many pots are there?"	
<u>Year 2</u> Children begin to relate division to fractions of numbers and shapes – e.g. ½ and ¼ is the same as dividing by 2 and dividing by 4 respectively. Children continue to use grouping of objects practically and relate to real life situations. They progress to grouping numbers into equal sets with a remainder. Teachers should introduce division as repeated subtraction (using a number line of bead bar). Then begin to divide a number by counting back in equal steps model this on a number line. Children begin to record their practical division as a written calculation using ÷ and = in a number sentence. Children learn that division is the inverse of multiplication. They are then taught to use the multiplication and division facts to work out missing numbers. e.g; 12 ÷ $\square$ = 4 Children use division to solve more complex word problems.	

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<tr>
<td>
<u>Year 3</u>

÷ = signs and missing numbers – using symbols to stand for unknown numbers to complete equations using inverse operations.

Continue using a range of equations as in Year 2, but with appropriate numbers.

Understand division with remainders.
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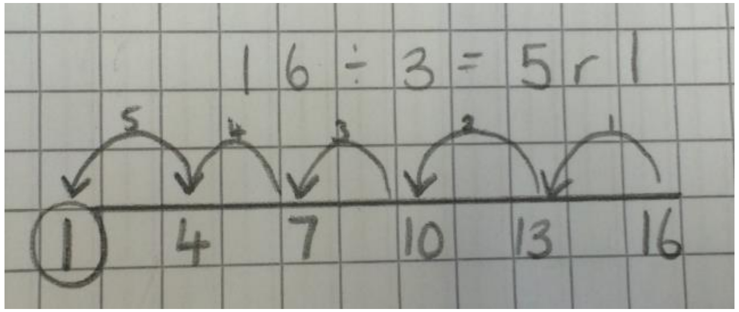
Sharing



</td>
</tr>
<tr>
<td>
<u>Year 4</u>

÷ = signs and missing numbers
Continue using a range of equations as in Year 2, but with appropriate numbers.
Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor. Initially, these should be multiples of 10s, 5s, 2s and 1s – numbers with which the children are more familiar.
72 ÷ 5
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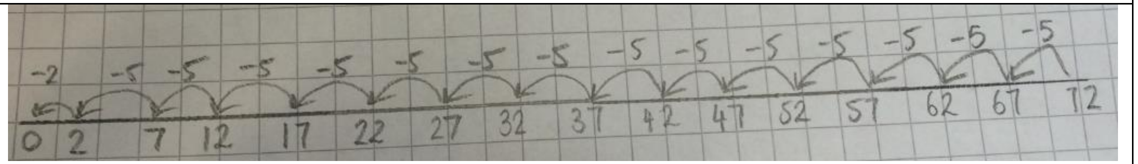
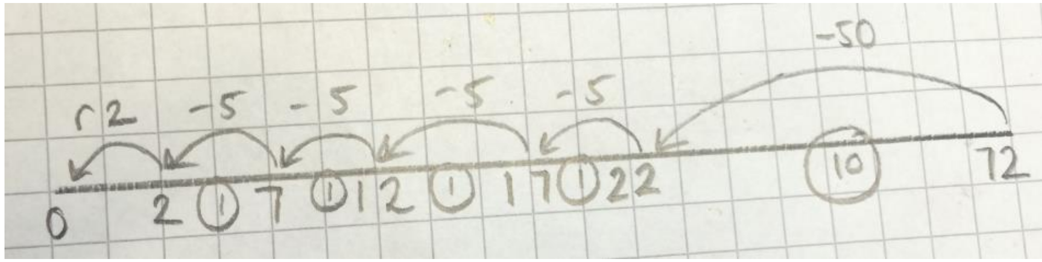


Leading to:



Leading to: vertical division (in preparation for chunking)
</td>
</tr>
</table>

Mental Division using partitioning

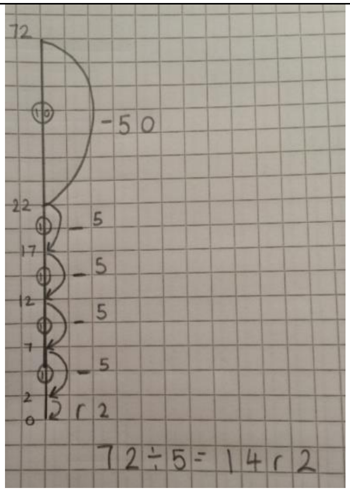
Children will be able to use mental strategies to divide. Mental methods for dividing TU ÷ U can be based on partitioning and on the distributive law of division over addition. This allows a multiple of the divisor and the remaining number to be divided separately. The results are then added to find the total quotient.

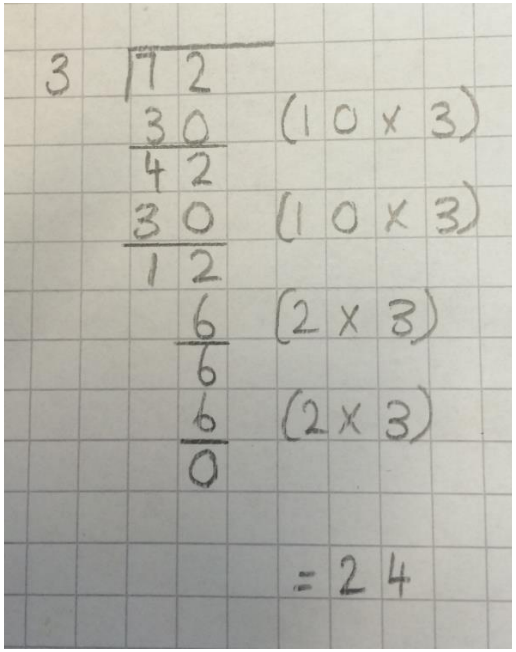
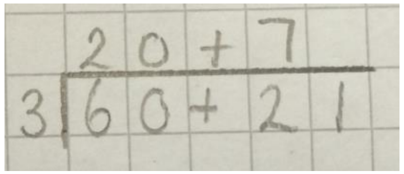
Many children can partition and multiply with confidence. But this is not the case for division. One reason for this may be that mental methods of division, stressing the correspondence to mental methods of multiplication, have not in the past been given enough attention.

Children should also be able to find a remainder mentally, for example the remainder when 34 is divided by 6.

One way to work out TU ÷ U mentally is to partition TU into a multiple of the divisor plus the remaining ones, then divide each part separately.

<u>Pencil and paper procedures – dividing two and three-digit</u>



numbers by a one-digit number Chunking for TU ÷ U 72 ÷ 3 Where 72 is the dividend and 3 is the divisor, this method is based on subtracting multiples of the divisor from the number to be divided, the dividend.	
Year 5 Once confident with chunking this should lead to: short division for TU ÷ U 'Short' division of TU ÷ U can be introduced as a more compact recording of the mental method of partitioning. Short division of a two-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound. For 81 ÷ 3, the dividend of 81 is split into 60, the highest multiple of 3 that is also a multiple 10 and less than 81, to give 60 + 21.	The short division method is recorded like this:  This is then shortened to:

Each number is then divided by 3.

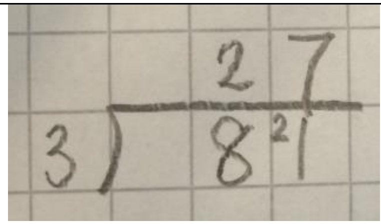
$$\begin{aligned} 81 \div 3 &= (60 + 21) \div 3 \\ &= (60 \div 3) + (21 \div 3) \\ &= 20 + 7 \\ &= 27 \end{aligned}$$

Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division.

For example 62 ÷ 8 is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context.
e.g. I have 62p. Sweets are 8p each. How many can I buy?
Answer: 7 (the remaining 6p is not enough to buy another sweet)

Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed?
Answer: 8 (the remaining 6 apples still need to be placed into a box)

<u>Pencil and paper procedures – dividing a three-digit or four-digit number by a one-digit number and a three-digit number by a two-digit number</u>



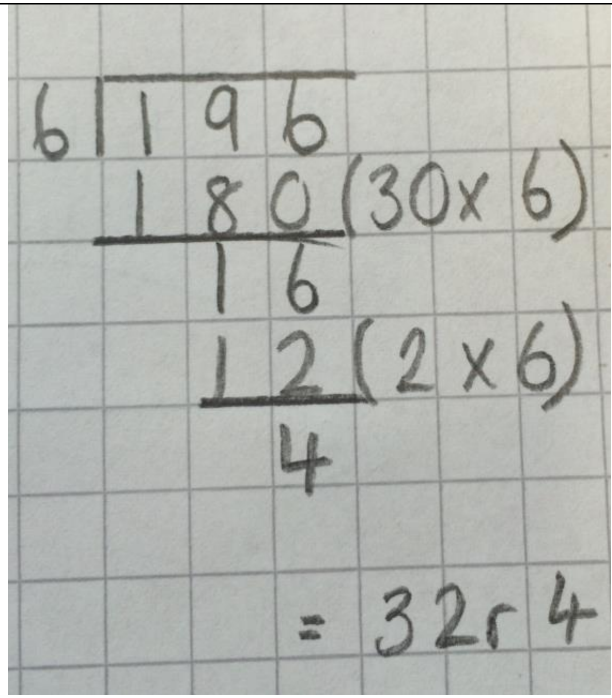
The carry digit ‘2’ represents the 2 tens that have been exchanged for 20 ones. In the first recording above it is written in front of the 1 to show that 21 is to be divided by 3. In second it is written as a superscript.

The 27 written above the line represents the answer: 20 + 7, or 2 tens and 7 ones.

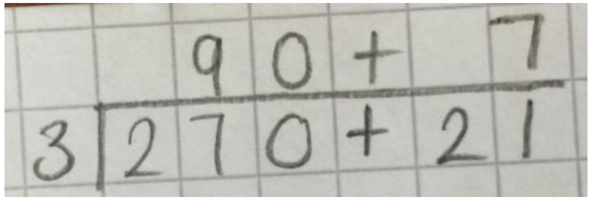
Children should start to subtract larger multiples of the divisor
196 ÷ 6

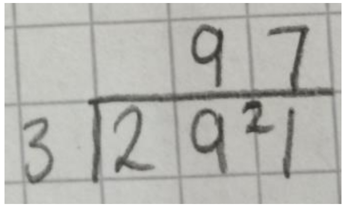
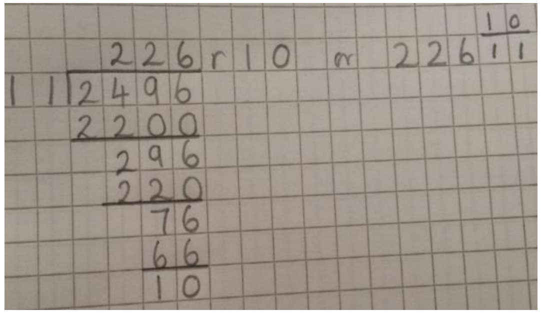
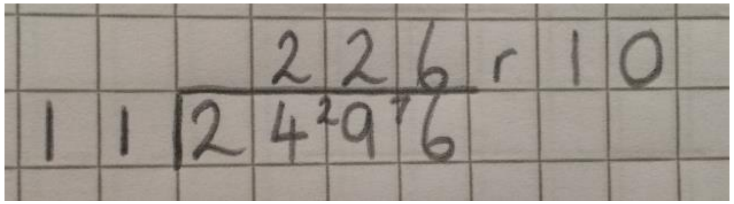
‘Short’ division of HTU ÷ U can be introduced as an alternative, more compact recording. No chunking is involved since the links are to partitioning, not repeated subtraction.

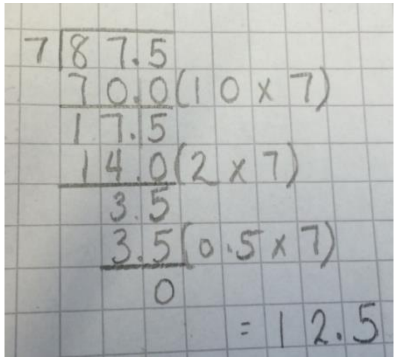
Quotients should be expressed as fractions or decimal fractions
61 ÷ 4 = 15 ¼ or 15.2



The short division method is recorded like this:



	This is then shortened to:  `The carry digit ‘2’ represents the 2 tens that have been exchanged for 20 ones. In the first recording above it is written in front of the 1 to show that a total of 21 ones are to be divided by 3. The 97 written above the line represents the answer: 90 + 7, or 9 tens and 7 ones.
<u>Year 6</u> <u>Pencil and paper procedures- Long Division – up to 4 digits by 2 digits.</u> 2496 divided by 11 <u>Pencil and paper procedures- Short Division– up to 4 digits by 2 digits.</u>	 

Extend to decimals with up to two decimal places. Children should know that decimal points line up under each other. 87.5 ÷ 7 Both methods above are necessary at this stage, to deal with the wide range of problems experienced at Year Six.	

By the end of year 6, children will have a range of calculation methods, mental and written. This will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

1) **they are not ready.**
2) **they are not confident.**

Children should be encouraged to approximate their answers before calculating.
They should be encouraged to check their answers after calculation using an appropriate strategy.
Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.